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# Humans Dental Arch Shapes 

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Abstract- The dental arches for humans change their shape from a parabola to an ellipse at canine. This was shown by the change in the value of discriminant $B^{2}-4 A C$ in the general second degree equation. In the present study the change for the dental arch shape has been alternatively suggested through the change in the value of eccentricity e, which is 1 for parabola and less than 1 for ellipse. The range of values between 0 and 1 for e can then be possibly assigned to different races or ethnic groups, through corresponding data values.

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# Humans Dental Arch Shapes 

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Abstract- The dental arches for humans change their shape from a parabola to an ellipse at canine. This was shown by the change in the value of discriminant $B^{2}-4 A C$ in the general second degree equation. In the present study the change for the dental arch shape has been alternatively suggested through the change in the value of eccentricity $e$, which is 1 for parabola and less than 1 for ellipse. The range of values between 0 and 1 for $e$ can then be possibly assigned to different races or ethnic groups, through corresponding data values.
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## I. Introduction

The dental arch shapes for humans and other mammals have been studied using the theory of conic sections. Attempts to provide mathematical description of dental arch shapes began during the first quarter of twentieth century (Bookstein 1984; Lestrel 1989). Mc Conail and Scher (1949), Scott (1957), Musich and Ackerman (1973) suggested Catenary curve as ideal curve for the dental arch. Lu (1964) claimed that the dental arch could be satisfactorily described by a fourth degree polynomial equation. Ramsden (1964) concluded that a parabola best represents an anterior curvature of the dental arch, although an arch that fitted a precise pattern was to be regarded as an exception rather than rule.

Currier (1969) found that the buccal surfaces of the maxillary arch conform more closely to an ellipse than to a parabola, but neither of these curves exhibited a significant fit to the lingual curves of the arches. Ridge (1981) reviewed the literature dealing with dental arch form and suggested that ideal arch as being constructed upon an equilateral triangle with slight modification. Ferrario et al (1994) used mixed elliptical plus parabolic model and concluded that shape of dental arch changes from an ellipse to a parabola at canine. Pokhariyal (1997) used a general second degree equation for the entire dental arch, with boundary condition at the canine, where parabola changes to an ellipse.

Pokhariyal et al. (2004) considered a general second degree equation

$$
\begin{equation*}
A x+B x y+C y^{2}+D x+E y+F=0 \tag{1}
\end{equation*}
$$

for the entire dental arch. The value of discriminant $B^{2}-4 A C$ equal to zero gives parabola, while less

[^0]than zero gives ellipse, the change of shape of dental arch at canine was verified by substituting the value of coordinates of the conic from parabola to an ellipse. The shapes were simulated for the data values from different ethnic groups, gender and the total sample and all shapes confirmed the change of parabola to ellipse at canine. This argument has been based on elementary knowledge of geometry, which suggests that the general second degree equation provides a mechanism for shifting form one curve to another at a point by fulfilling the required conditions. It was further shown that the inner region of the dental arch remained parabolic throughout, without changing from parabola to ellipse at canine.

## iI. Alternative Representation

In this study, an alternative definition for all conics is provided from the basic concepts of geometry, which can then be used to represent dental arch shape for humans, that change from one conic to another. "A conic is the set of points located in a plane such that the undirected distance of each point form a fixed point divided by its undirected distance from a fixed line is the same constant". The fixed point is called the focus, the fixed line is known as the directrix and the constant $e$ is referred to as the eccentricity.

The equation of the conic with eccentricity e, a focus at $(p, o)$ and directrix along the $y$-axis is

$$
\begin{equation*}
\left(1-e^{2}\right) x^{2}+y^{2}-2 x p+p^{2}=0 \tag{2}
\end{equation*}
$$



Figure 1a: Conics with y-axis as directrix
Similarly the equation of conic with eccentricity $e$, a focus at $(0, p)$ and directrix along the $x$-axis is

$$
\begin{equation*}
\left(1-e^{2}\right) y^{2}+x^{2}-2 p y+p^{2}=0 . \tag{3}
\end{equation*}
$$



Figure 1b: Conic with $x$-axis as directrix.


Figure 2a: Human dental arch.

## III. Discussion

The dental arch shape can therefore be determined through change in the value of discriminant or eccentricity at the canine from parabola to ellipse. Hence, there seems to be no need to make ad-hock assumptions about the dental arch shape. There can be infinite numbers between 0 and 1 , so each Individual can have unique value of $e$ for their dental arch shape. Thus, in the classification of dental arch shapes (that change at canine) for different races, ethnic groups and gender, the values of eccentricity can be computed. The range of values between 0 and 1 can then be assigned to these groups, through the analysis of the respective data values.

It is understood that dental arch portion which is parabolic (up to canine) influences the shape of small part of upper portion of chin and lips. It is considered that middle and upper base of the face is governed by the dental arch which is ellipse. Thus, one could possibly link the facial base with the corresponding values of $e$ for different racial and ethnic groups. Such findings can then be used for relatively accurate arch-

According to the value of eccentricity $e$, the conics are classified in a manner that for ellipse e lies between 0 and 1 . The extreme values 0 and 1 give circle and parabola respectively.

The change of the shape at canine for human data values can thus be confirmed for the collected data values from the subjects, using these conditions.


Figure 2b: Change of parabola to ellipse at canine.
wire construction and preparation on dental prostheses. In future, the study of change in dental arch shape can also be extended to other mammals.

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