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Thermodynamical Behavior of Laser Irradiated Mass Diffusive Micro Stretch Thermoelastic Medium

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I. Introduction

Eringen [1] developed the theory of thermo-micro stretch elastic solids. Micro stretch continuum is a model for Bravais lattice with basis on the atomic level and two-phase dipolar solids with a core on the macroscopic level. Composite materials reinforced with chopped elastic fibers, porous media with pores containing gas or in viscid liquid, asphalt or other elastic inclusions and solid-liquid crystals etc. are examples of micro stretch solids. Ezzat et al. [2, 3] discussed the concept of thermal relaxation. Marin [4, 5] investigated various problems in micropolar thermoelasticity and micro stretch thermoelasticity.

Diffusion is the spontaneous movement of the particles from a high concentration region to the low-concentration region, and it occurs in response to a concentration gradient expressed as the change in the concentration due to change in position. Thermal diffusion utilizes the transfer of heat across a thin liquid or gas to accomplish isotope separation. Simply concentration is calculated using Fick's law. This law does not consider the mutual interaction between the inclusion substance and the medium. The thermo diffusion in elasticity is result of the coupling of temperature, mass diffusion and that of strain in addition to heat and mass exchange with the environment. Nowacki [6-9] developed the theory of thermoelastic diffusion by using coupled thermoelastic model. Dudziak and Kowalski [10] and Olesiak and Pyryev [11], respectively, discussed the theory of thermo diffusion and coupled quasi-stationary problems of thermal diffusion for an elastic layer.

Thermal shock due to exposure to an ultra-short laser pulse is interesting from the point of thermo elasticity since they require a coupled analysis of the temperature and deformation fields. A thermal shock induces very rapid movement in the structural elements, giving rise to very significant inertial forces, and thereby, an increase in vibration. In irradiation of ultra-short pulsed laser, the high-intensity energy flux and ultra-short duration lead to a very high thermal gradient. So, in these cases, Fourier law of heating is no longer valid. Scruby et al. [12] and Rose [13] considered the point source model of lasers. Later McDonald [14] and Spicer [15] proposed a new model known as laser-generated ultrasound model by introducing the thermal diffusion effect. Dubois [16] experimentally demonstrated that penetration depth plays an important role in the laser-ultrasound generation process. The thermoelastic response of laser in context of four theories was discussed by Youssef and Al-Bary [17]. A problem for a thick plate under the effect of laser pulse thermal heating was studied by Elhagary [18]. Kumar et al. [19] studied the thermo-mechanical interactions of a laser pulse with the micro stretch thermoelastic medium.

This present research deals with the disturbance in a homogeneous micro stretch thermoelastic medium with mass diffusion due to the effect of an ultra-laser heat source. The normal mode analysis technique is used to obtain the expressions for the displacement components, couple stress, temperature, mass concentration and micro stress distribution due to various sources.

II. Basic Equations

Following Eringen [20] and Al-Qahtani and Datta [21], the basic equations for homogeneous micro stretch thermoelastic mass diffusion medium in the absence of body force, body couple with laser heat source are given by:

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a) *Stress equation of motion*

$$(\lambda+\mu)\nabla(\nabla.u)+(\mu+K)\nabla^2 u+K\nabla\times\phi+\lambda_0\nabla\phi^*-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T-\beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla C=\rho\ddot{u} \tag{1}$$

b) *Couple stress equation of motion*

$$(\gamma\nabla^2-2K)\phi+(\alpha+\beta)\nabla(\nabla.\phi)+K\nabla\times u=\rho j\ddot{\phi} \tag{2}$$

c) *The equation of balance of stress moments*

$$(\alpha_0\nabla^2-\lambda_1)\phi^*-\lambda_0\nabla.u+\nu_1\left(1+\tau_1\frac{\partial}{\partial t}\right)T+\nu_2\left(1+\tau^1\frac{\partial}{\partial t}\right)C=\frac{\rho j_0}{2}\ddot{\phi}^* \tag{3}$$

d) *The equation of heat conduction*

$$K^*\nabla^2 T=\rho c^*\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T+\beta_1 T_0\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)(\nabla.u-Q)+\nu_1 T_0\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)\phi^*+aT_0\left(\frac{\partial}{\partial t}+\gamma_1\frac{\partial^2}{\partial t^2}\right)C \tag{4}$$

e) *The equation of mass diffusion is*

$$D\beta_2\nabla^2(\nabla.u)+Da\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla^2 T+\left(\frac{\partial}{\partial t}+\varepsilon\tau^0\frac{\partial^2}{\partial t^2}\right)C-Db\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla^2 C=0 \tag{5}$$

f) *The constitutive relations are*

$$t_{ij}=(\lambda_0\phi^*+\lambda u_{r,r})\delta_{ij}+\mu(u_{i,j}+u_{j,i})+K(u_{j,i}-\epsilon_{ijk}\phi_k)-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\delta_{ij}T-\beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)\delta_{ij}C \tag{6}$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}+b_0\epsilon_{mji}\phi^*_{,m} \tag{7}$$

$$\lambda_i^*=\alpha_0\phi^*_{,i}+b_0\epsilon_{ijm}\phi_{j,m} \tag{8}$$

The plate surface is illuminated by laser pulse given by the heat input

$$Q=I_0 f(t)g(x_1)h(x_3) \tag{9}$$

Where I_0 is the energy absorbed. The temporal profile $f(t)$ is represented as,

$$f(t)=\frac{t}{t_0^2}e^{-\left(\frac{t}{t_0}\right)} \tag{10}$$

Here t_0 is the pulse rise time. The pulse is also assumed to have a Gaussian spatial profile in x_1

$$g(x_1)=\frac{1}{2\pi r^2}e^{-\left(\frac{x_1^2}{r^2}\right)} \tag{11}$$

Where r is the beam radius, and as a function of the depth x_3 the heat deposition due to the laser pulse is assumed to decay exponentially within the solid,

$$h(x_3)=\gamma^* e^{-\gamma^* x_3} \tag{12}$$

Equation (9) with the aid of (10-11) and (12) takes the form:

$$Q=\frac{I_0\gamma^*}{2\pi r^2 t_0^2}te^{-\left(\frac{t}{t_0}\right)}e^{-\left(\frac{x_1^2}{r^2}\right)}e^{-\gamma^* x_3}, \tag{13}$$

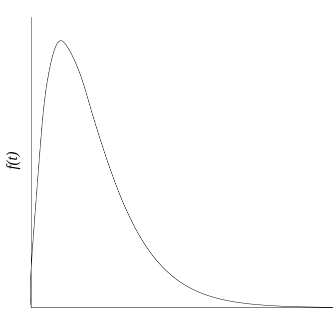

Fig. 1: Temporal profile of $f(t)$.

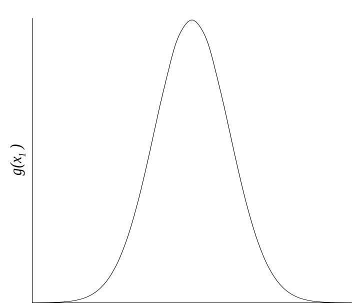

Fig. 2: Profile of $g(x_1)$.

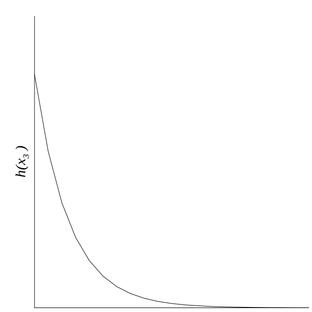

Fig. 3: Profile of $h(x_3)$.

Here $\lambda,\ \mu,\ \alpha,\ \beta,\ \gamma,\ K, \lambda_0, \lambda_1, \alpha_0, b_0$, are material constants, ρ is mass density, $u = (u_1, u_2, u_3)$is the displacement vector and $\phi = (\phi_1, \phi_2, \phi_3)$is the microrotation vector, $\phi^*$is the scalar micro stretch function, T is temperature and T_0 is the reference temperature of the body chosen,C is the concentration of the diffusion material in the elastic body,K^* is the coefficient of the thermal conductivity,c^* is the specific heat at constant strain, D is the thermoelastic diffusion constant, a is the coefficient describing the measure of thermo diffusion and b is the coefficient describing the measure of mass diffusion effects, j is the microinertia, $\beta_1 = (3\lambda + 2\mu + K)\alpha_{t1}$, $\beta_2 = (3\lambda + 2\mu + K)\alpha_{c1}$, $\nu_1 = (3\lambda + 2\mu + K)\alpha_{t2}$, $\nu_2 = (3\lambda + 2\mu + K)\alpha_{c2}$, α_{t1}, α_{t2} are coefficients of linear thermal expansion and α_{c1}, α_{c2} are coefficients of linear diffusion expansion, j_0 is the microinertia for the microelements,t_{ij} are components of stress, m_{ij} are components of couple stress, λ_i^* is the micro stress tensor, e_{ij} are components of strain, e_{kk} is the dilatation,δ_{ij} is Kroneker delta function, τ^0, τ^1 are the diffusion relaxation times and $\tau_0, \tau_1$are thermal relaxation times with $\tau_0 \geq \tau_1 \geq 0$.

In the above equations symbol (",") followed by a suffix denotes differentiation with respect to spatial coordinates and a superposed dot (" $\cdot$ ") denotes the derivative with respect to time respectively.

III. Formulation of the Problem

We consider a micro stretch thermoelastic mass diffusion medium with rectangular Cartesian coordinate system $OX_1X_2X_3$ with x_3-axis pointing vertically downward the medium.

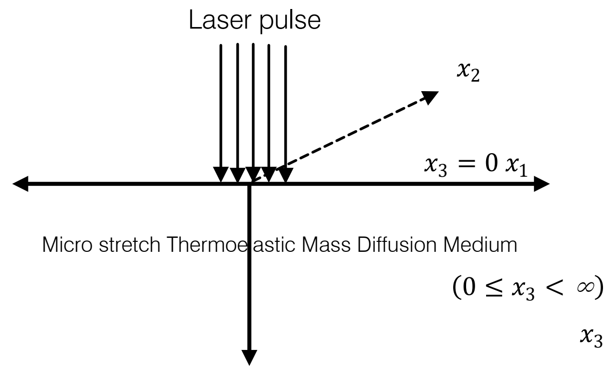

Fig. 4: Geometry of the Problem

For two dimensional problems, we take the displacement vector and micro rotation vector as:

$$u = (u_1, 0, u_3), \phi = (0, \phi_2, 0), \tag{14}$$

For further consideration it is convenient to introduce in equations (1)-(5) the dimensionless quantities defined by:

$$u_i' = \frac{\rho\omega^* c_1}{\beta_1 T_0} u_i\,,\ x_i' = \frac{\omega^*}{c_1} x_i\,,\ t' = \omega^* t\,,\ T' = \frac{T}{T_0}\,,\ \tau_1' = \omega^*\tau_1\,,\ \tau_0' = \omega^*\tau_0\,, \gamma_1' = \omega^*\gamma_1,\ t_{ij}' = \frac{1}{\beta_1 T_0} t_{ij}\,,\ \omega^* = \frac{\rho c^* c_1^2}{K^*}, \phi_i' = \frac{\rho c_1^2}{\beta_1 T_0}\phi_i\,,$$
$$\tau^{1'} = \omega^*\tau^1\,,\ c_1^2 = \frac{\lambda+2\mu+k}{\rho}\,,\ c_2^2 = \frac{\mu+k}{\rho}\,,\ c_3^2 = \frac{\gamma}{\rho j}\,,\ c_4^2 = \frac{2\alpha_0}{\rho j_0}\,,\ \varepsilon = \frac{\gamma^2 T_0}{\rho^2 c^* c_1}\,, m_{ij}^* = \frac{\omega^*}{c\beta_1 T_0} m_{ij}\ ,\ C' = \frac{\beta_2}{\rho c_1^2} C\,,\ Q = \frac{K^*\omega^{*2}}{C^*} Q', \phi^{*'} = \frac{\rho c_1^2}{\beta_1 T_0}\phi^* \tag{15}$$

By Helmholtz representation of a vector into scaler and vector potentials the displacement components u_1 and u_3 are related to non- dimensional potential functions ϕ and ψ as:

$$u_1 = \frac{\partial\phi}{\partial x_1} - \frac{\partial\psi}{\partial x_3}\ ,\quad u_3 = \frac{\partial\phi}{\partial x_3} + \frac{\partial\psi}{\partial x_1} \tag{16}$$

Substituting the values of $u_1 \& u_3$ from (16) in (1)-(5) and with the aid of (14) & (15), after suppressing the primes, we obtain:

$$\nabla^2\phi - \ddot{\phi} + a_4\phi^* - \left(1 + \tau_1\frac{\partial}{\partial t}\right)T - a_5\left(1 + \tau^1\frac{\partial}{\partial t}\right)C = 0, \tag{17}$$

$$\left(\nabla^2 - a_8 - a_{12}\frac{\partial^2}{\partial t^2}\right)\phi^* - a_9\nabla^2\phi + a_{10}\left(1 + \tau_1\frac{\partial}{\partial t}\right)T + a_{11}\left(1 + \tau^1\frac{\partial}{\partial t}\right)C = 0, \tag{18}$$

$$\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2} - \nabla^2\right)T + \left(1 + \varepsilon\tau_0\frac{\partial}{\partial t}\right)\left(a_{13}\nabla^2\phi + a_{14}\dot{\phi}^*\right) + a_{15}\left(1 + \gamma_1\frac{\partial}{\partial t}\right)\dot{C} = Q_0 f^*(x_1, t)e^{-\gamma^* x_3}, \tag{19}$$

$$\nabla^4\phi + a_{16}\left(1 + \tau_1\frac{\partial}{\partial t}\right)\nabla^2 T + a_{17}\left(\frac{\partial}{\partial t} + \varepsilon\tau^0\frac{\partial^2}{\partial t^2}\right)C - a_{18}\left(1 + \tau^1\frac{\partial}{\partial t}\right)\nabla^2 C = 0 \tag{20}$$

$$a_2\nabla^2\psi - \ddot{\psi} + a_3\phi_2 = 0, \tag{21}$$

$$\nabla^2\phi_2 - 2a_6\phi_2 - a_6\nabla^2\psi = a_7\ddot{\phi}_2, \tag{22}$$

Here$\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}$ is Laplacian operator, $f(x_1, t) = \left[t + \epsilon\tau_0\left(1 - \frac{t}{t_0}\right)\right]e^{-\left(\frac{x_1^2}{r^2} + \frac{t}{t_0}\right)}$ and $Q_0 = \frac{Q_{20} I_0 \gamma^*}{2\pi r^2 t_0^2}$

IV. Solution of the Problem

The solution of the considered physical variables can be decomposed in terms of the normal modes as in the following form:

$$\{\phi,\psi,T,\phi_2,\phi^*,C\}(x_1,x_3,t)=\{\bar{\phi},\bar{\psi},\bar{T},\overline{\phi_2},\overline{\phi^*},\bar{C}\}(x_3)e^{i(kx_1-\omega t)} \tag{23}$$

Here ω is the angular frequency and k is wave number.

Making use of (23), equations (17)-(22) after some simplifications yield:

$$[AD^8+BD^6+CD^4+ED^2+F]\bar{\phi}=f_1(\gamma^*,x_1,t)e^{-\gamma^*x_3} \tag{24}$$

$$[AD^8+BD^6+CD^4+ED^2+F]\overline{\phi^*}=f_2(\gamma^*,x_1,t)e^{-\gamma^*x_3} \tag{25}$$

$$[AD^8+BD^6+CD^4+ED^2+F]\bar{T}=f_3(\gamma^*,x_1,t)e^{-\gamma^*x_3} \tag{26}$$

$$[AD^8+BD^6+CD^4+ED^2+F]\bar{C}=f_4(\gamma^*,x_1,t)e^{-\gamma^*x_3} \tag{27}$$

$$[D^4+GD^2+H]\bar{\psi}=0 \tag{28}$$

Where $D=\frac{d}{dx_3}$, $A=a_{21}-a_{33}, B=a_{37}-2k^2a_{21}-a_{31}a_{39}+-a_{34},$

$C=a_{38}+a_{21}k^4-2k^2a_{37}-a_{32}a_{39}-a_{31}a_{40}+a_{33}a_{43}+a_{34}a_{42}$, $H=-(k^2a_3a_6+a_{35}a_{36})/a_2$

$E=a_{37}k^4-2k^2a_{38}-a_{32}a_{40}-a_{31}a_{41}, F=a_{38}k^4-a_{32}a_{41}+a_{34}a_{44}, G=a_{35}+a_3a_6-a_2a_{36}/a_2,$

Also, a_i , $i=19,\dots,44$ are defined in appendix A.

The solution of the above system of equations (24)-(28) satisfying the radiation conditions that $(\bar{\phi},\bar{\psi},\bar{T},\overline{\phi_2},\bar{C})\to 0$ as $x_3\to\infty$ are given as following:

$$\bar{\phi}=\sum_{i=1}^{4}c_ie^{-m_ix_3}+\frac{f_1}{f_5}e^{-\gamma^*x_3} \tag{29}$$

$$\overline{\phi^*}=\sum_{i=1}^{4}\alpha_{1i}c_ie^{-m_ix_3}+\frac{f_2}{f_5}e^{-\gamma^*x_3} \tag{30}$$

$$\bar{T}=\sum_{i=1}^{4}\alpha_{2i}c_ie^{-m_ix_3}+\frac{f_3}{f_5}e^{-\gamma^*x_3} \tag{31}$$

$$\bar{C}=\sum_{i=1}^{4}\alpha_{3i}c_ie^{-m_ix_3}+\frac{f_4}{f_5}e^{-\gamma^*x_3}, \tag{32}$$

$$(\bar{\psi},\overline{\phi_2})=\sum_{i=5}^{6}(1,\delta_i)c_ie^{-m_ix_3}, \tag{33}$$

Where$m_i^2(i=1,2,3,4)$ are the roots of the equation (24) and $m_i^2(i=5,6)$ are the roots of characteristic equation of equation (28) and

$$\alpha_{1i}=-\frac{\Delta_{2i}}{\Delta_{1i}},\alpha_{2i}=\frac{\Delta_{3i}}{\Delta_{1i}},\alpha_{3i}=-\frac{\Delta_{4i}}{\Delta_{1i}},\ i=1,2,3,4\&\delta_i=\frac{a_3}{(a_2m_i^2+a_{35})},\ i=5,6$$

Here, $\Delta_{1i},\Delta_{2i},\Delta_{3i},\Delta_{4i}$ are defined in Appendix B.

Substituting the values of $\bar{\phi},\overline{\phi^*},\bar{T},\bar{\psi},\bar{\phi}_2,\bar{C}$ from the equations (29)-(33) in the (6)-(8), and using (14)-(16) & (23) and then solving the resulting equations, we obtain:

$$\bar{t}_{33}=\sum_{i=1}^{6}G_{1i}e^{-m_ix_3}-M_1e^{-\gamma^*x_3} \tag{34}$$

$$\bar{t}_{31}=\sum_{i=1}^{6}G_{2i}e^{-m_ix_3}-M_2e^{-\gamma^*x_3}, \tag{35}$$

$$\bar{m}_{32}=\sum_{i=1}^{6}G_{3i}e^{-m_ix_3}-M_3e^{-\gamma^*x_3} \tag{36}$$

$$\lambda_3^*=\sum_{i=1}^{6}G_{4i}e^{-m_ix_3}-M_4e^{-\gamma^*x_3}, \tag{37}$$

$$\bar{T}=\sum_{i=1}^{6}G_{5i}e^{-m_ix_3}-M_5e^{-\gamma^*x_3}, \tag{38}$$

$$\bar{C}=\sum_{i=1}^{5}G_{6i}e^{-m_ix_3}-M_6e^{-\gamma^*x_3}, \tag{39}$$

$G_{mi}=g_{mi}C_i$, $i,m=1,2,\dots,6$. G_{rs} , $(r,s=1,2,\dots,6)$,M_r , $(r=1,2,\dots,6)$ are described in Appendix C.

V. Boundary Conditions

We consider normal force and thermal and mass concentration sources are acting at the surface $x_3 = 0$ along with vanishing of couple stress in addition to thermal and mass concentration boundaries considered at$x_3 = 0$ and$I_0 = 0$. Mathematically this can be written as:

$$t_{33} = -F_1 e^{i(kx_1-\omega t)},\ t_{31} = 0,\ m_{32} = 0,\ \lambda_3^* = 0,\ \frac{\partial T}{\partial x_3} = F_2 e^{i(kx_1-\omega t)},\ \frac{\partial C}{\partial x_3} = F_3 e^{i(kx_1-\omega t)} \quad (40)$$

Where F_1 and F_2 are the magnitude of the applied force.

Substituting the expression of the variables considered into these boundary conditions, we can obtain the following system of equations:

$$\sum_{i=1}^{6}(G_{1i}, G_{2i}, G_{3i}, G_{4i}, m_i G_{5i}, m_i G_{6i})c_i = (-F_1, 0,0,0, -F_2, -F_3) \quad (41)$$

The system of equations (41) is solved by using the matrix method as follows:

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} & g_{15} & g_{16} \\ g_{21} & g_{22} & g_{23} & g_{24} & g_{25} & g_{26} \\ g_{31} & g_{32} & g_{33} & g_{34} & g_{35} & g_{36} \\ g_{41} & g_{42} & g_{43} & g_{44} & g_{45} & g_{46} \\ m_1 g_{51} & m_2 g_{52} & m_3 g_{53} & m_4 g_{54} & m_5 g_{55} & m_6 g_{56} \\ m_1 g_{61} & m_2 g_{62} & m_3 g_{63} & m_4 g_{64} & m_5 g_{65} & m_6 g_{66} \end{bmatrix}^{-1} \begin{bmatrix} -F_1 \\ 0 \\ 0 \\ 0 \\ -F_2 \\ -F_3 \end{bmatrix} \quad (42)$$

VI. Special Cases

a) Micro stretch Thermoelastic Solid

If we neglect the diffusion effect in (41), we obtain the corresponding expressions of stresses, displacements and temperature for micro stretch thermoelastic solid.

b) Micropolar Thermoelastic Diffusive Solid

If we neglect the micro stretch effect in (41), we obtain the corresponding expressions of stresses, displacements and temperature for micropolar thermoelastic diffusive solid.

VII. Numerical Results and Discussions

The analysis is conducted for a magnesium crystal-like material. The values of constants are as:

$$\lambda = 9.4 \times 10^{10} Nm^{-2}, \mu = 4.0 \times 10^{10} Nm^{-2},\ K = 1.0 \times 10^{10} Nm^{-2},\ \rho = 1.74 \times 10^3 Kgm^{-3}, j = 0.2 \times 10^{-19} m^2,\ \gamma = 0.779 \times 10^{-9} N$$

Thermal, diffusion and micro stretch parameters are given by:

$$c^* = 1.04 \times 10^3 JKg^{-1}K^{-1},$$
$$K^* = 1.7 \times 10^6 Jm^{-1}s^{-1}K^{-1}, \alpha_{t1} = 2.33 \times 10^{-5} K^{-1}, \alpha_{t2} = 2.48 \times 10^{-5} K^{-1}, T_0 = 0.298 \times 10^3 K, \tau_0 = 0.02, \tau_1 = 0.01, \alpha_{c1} = 2.65 \times 10^{-4} m^3 Kg^{-1}, \alpha_{c2} = 2.83 \times 10^{-4} m^3 Kg^{-1}, a = 2.9 \times 10^4 m^2 s^{-2} K^{-1}, b = 32 \times 10^5 Kg^{-1} m^5 s^{-2}, \tau^1 = 0.04, \tau^0 = 0.03, D = 0.85 \times 10^{-8} Kgm^{-3} s,$$
$$j_0 = 0.19 \times 10^{-19} m^2,\ \alpha_0 = 0.779 \times 10^{-9} N,\ b_0 = 0.5 \times 10^{-9} N,\ \lambda_0 = 0.5 \times 10^{10} Nm^{-2}, \lambda_1 = 0.5 \times 10^{10} Nm^{-2}$$

A comparison of the dimensionless form of the field variables for the cases of micro stretch thermoelastic mass diffusion medium with a laser pulse (MTMDL), micro stretch thermoelastic mass diffusion medium without a laser pulse (MTMD) subjected to normal force is presented in Figures 5-13. The values of all physical quantities for both cases are shown in the range $0 \le x_3 \le 5$.

Solid lines, dash lines corresponds to micro stretch thermoelastic mass diffusion medium with laser pulse (MTMDL) and micro stretch thermoelastic mass diffusion medium without laser pulse (MTMD) respectively. The computations were carried out in the absence and presence of laser pulse ($I_0 = 10^5, 0$) and on the surface of plane $x_1 = 1, t = 0.1$

Fig. 5 shows the variation of normal stress t_{33} with the distance x_3. It is noticed that for MTMDL and MTMD, the normal stress t_{33} show similar behavior. The normal stress in both the cases initially increases and then monotonically decreases. The value of t_{33} increases near the application of the normal force due to the stretch effect and then decreases.

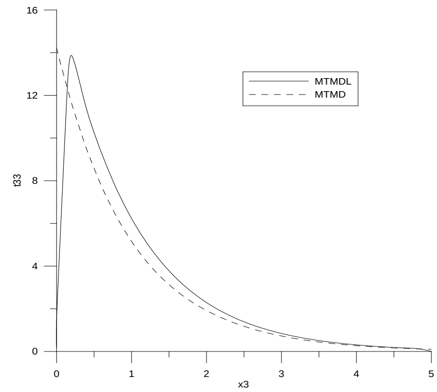

Fig. 5: Variation of normal stress

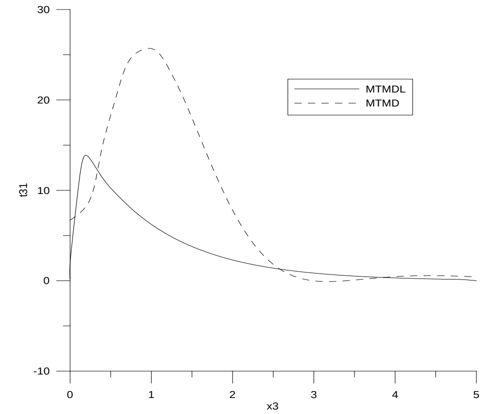

Fig. 6: Variation of tangential stress

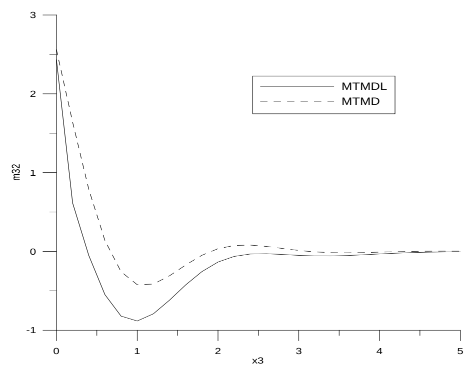

Fig. 7: Variation of coupled tangential stress

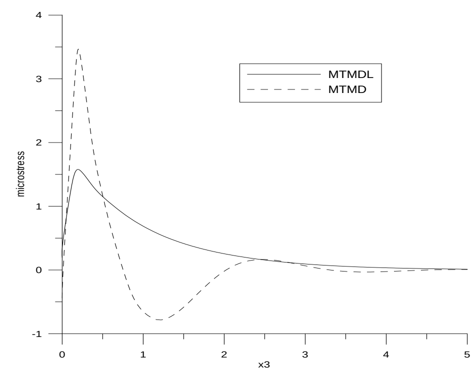

Fig. 8: Variation of micro stress

Fig. 6 displays the variation of tangential stress t_{31} with the distancex_3. It is noticed that initially the behavior of t_{31} for MTMDL and MTMD is similar. Initially t_{31} increases monotonically for MTMDL and MTMD and then approaches to the boundary surface away from the point of application of normal force.

Fig. 7 shows the variation of couple stress m_{32} with distance x_3 for MTMDL and MTMD. The variation of m_{32} for (MTMDL, MTMD) is monotonically decreasing in region $0 \leq x_3 \leq 1$ and monotonically increasing after that. The m_{32} approaches to zero away from the point of application of source. It is clear from figure 3 that laser source has a significant effect on the value ofm_{32}.

Fig. 8 depicts the variation of micro stress λ_3^* with distance. The variation of λ_3^* is similar for both the cases in the beginning and in the last, however λ_3^* for MTMD show oscillatory behavior in range$1 \leq x_3 \leq 4$

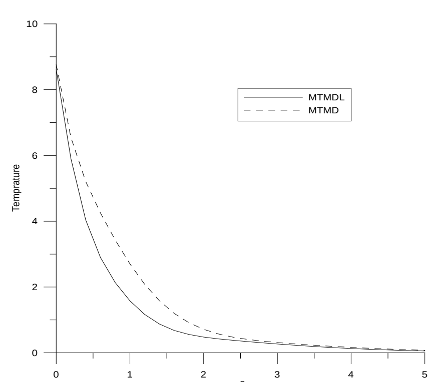

Fig. 9: Variation of temperature distribution

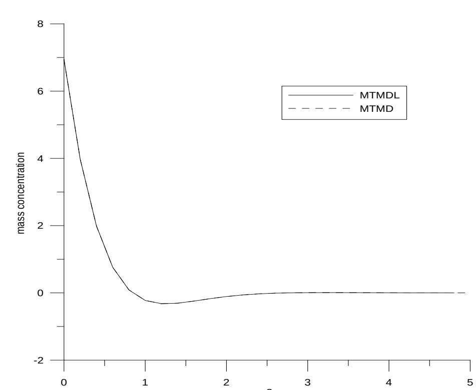

Fig. 10: Variation of mass concentration

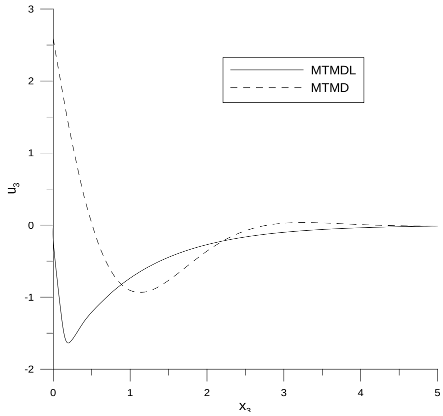

Fig.11: Variation of u_1 w.r.t. x_3.

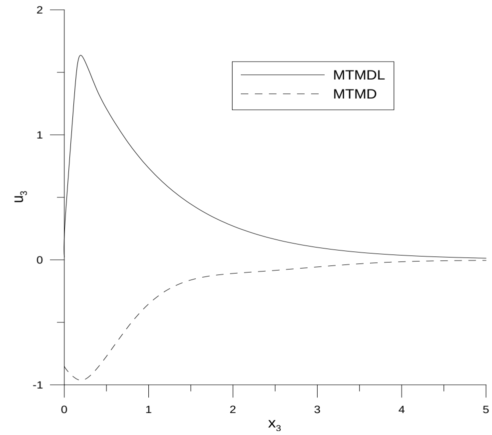

Fig. 12: Variation of u_3 w.r.t. x_3.

Fig. 9 displays the variation of temperature T with distancex_3. The values of temperature change for MTMDL and MTMD show monotonically decreasing behavior in the range $0 \leq x_3 \leq 5$. In case of MTMDL the temperature decreases more rapidly in comparison to MTMD due to the presence of input ultra-short laser heat source.

Fig. 10 show variation of mass concentration w.r.t. distancex_3. Mass concentration monotonically decreases with increasing distance from application of source. The laser source seems to have no significant effect on variation of mass concentration.

Fig. 11 and Fig.12 exhibits the behavior of displacement components u_1 and u_3w.r.t. x_3. Both the displacement components approaches to boundary surface away from the application of normal force which is in agreement to the generalized theory of thermoelasticity.

VIII. Variation of Temperature with Respect to Time

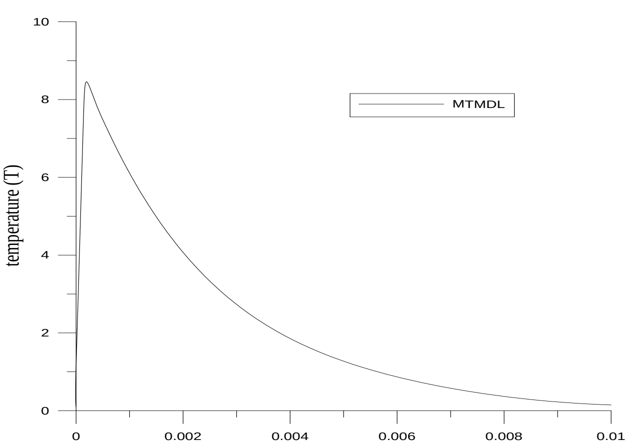

Fig. 13: Variation of temperature w.r.t. time

Fig. 13 represent the variation of temperature distribution w.r.t. time. As the laser is irradiated the temperature increases rapidly. The temperature decreases uniformly after reaching a peak value.

IX. Conclusions

The problem consists of investigating displacement components, scalar micro stretch, temperature distribution and stress components in a micro stretch thermoelastic mass diffusion medium subjected to input laser heat source. Normal mode analysis is employed to express the results. Theoretically obtained field variables are also depicted graphically.

The analysis of results permits some concluding remarks:

1) It is clear from the figures that all the field variables have nonzero values only in the bounded region of space indicating that all the results are in agreement with the various theories of thermoelasticity.
2) The effect of the input laser heat source is much pronounced in normal stress, tangential stress, micro stress, temperature distribution and displacement components. Change in the value of $\boldsymbol{I_0}$ cause significant changes in all these simulated resulting quantities.
3) It is noticed from the figures that the laser heat source has no significant role on mass concentration.
4) The trend of variation of physical quantities show similarity with Elhagary [18] although diffusion effect is included.

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Appendix A

$$a_1 = \frac{\lambda+\mu}{\beta_1 T_0}, a_2 = \frac{\mu+K}{\beta_1 T_0}, a_3 = \frac{K}{\rho c_1^2}, a_4 = \frac{\lambda_0}{\rho c_1^2}, a_5 = \frac{\rho c_1^2}{\beta_1 T_0}, a_6 = \frac{K c_1^2}{\gamma \omega^{*2}}, a_7 = \frac{\rho j c_1^2}{\gamma}, a_8 = \frac{\lambda_1 c_1^2}{\alpha_0 \omega^{*2}}, a_9 = \frac{\lambda_0 \rho c_1^4}{\beta_1 T_0 \alpha_0 \omega^{*2}}, a_{10} = \frac{\upsilon_1 \rho c_1^4}{\beta_1 \alpha_0 \omega^{*2}}, a_{11} = \frac{\nu_2 \rho^2 c_1^6}{\beta_1 \beta_2 T_0 \alpha_0 \omega^{*2}}, a_{12} = \frac{\rho c_1^2 j_0}{2\alpha_0}, a_{13} = \frac{T_0 \beta_1^2}{\rho \omega^* K^*}, a_{14} = \frac{\upsilon_1 \beta_1 T_0}{\rho \omega^* K^*}, a_{15} = \frac{a \rho c_1^4}{\omega^* \beta_2 K^*}, a_{16} = \frac{a \rho c_1^2}{\beta_1 \beta_2}, a_{17} = \frac{\rho c_1^4}{\omega^* D \beta_2^2}, a_{18} = \frac{b \rho c_1^2}{\beta_2^2}, Q_{20} = \frac{\rho c_1^4}{\omega^{*2} \beta_1 K^*}, a_{19} = \omega^2 - k^2,$$

$$a_{20} = 1 - i\omega\tau_1, a_{21} = a_5(1 - i\omega\tau), a_{22} = k^2 a_9, a_{23} = \omega^2 a_{12} - a_8 - k^2, a_{24} = a_{10}(1 - i\omega\tau_1), a_{25} = a_{11}(1 - i\omega\tau'), a_{26} = -a_{13}(i\omega + \omega^2 \varepsilon\tau_0), a_{27} = k^2 a_{26}, a_{32} = -k^2 a_{31}, a_{28} = -a_{14}(i\omega + \omega^2 \varepsilon\tau_0), a_{29} = k^2 - i\omega - \omega^2\tau_0, a_{30} = -a_{15}(i\omega + \omega^2\gamma_1), a_{31} = a_{16}(1 - i\omega\tau_1), a_{33} = -a_{18}(1 - i\omega\tau'), a_{34} = a_{17}(i\omega - \omega^2 \varepsilon\tau^0), a_{35} = \omega^2 - k^2 a_2, a_{36} = \omega^2 a_7 - k^2 - 2a_6, a_{37} = a_4 a_{25} + a_{20} a_{30} - a_{29} + a_{23}, a_{38} = a_4(a_{24}a_{30} - a_{29}a_{25}) + a_4(a_{24}a_{30} - a_{29}a_{25}) - a_4(a_{24}a_{30} - a_{29}a_{25}), a_{39} = a_{21}a_{26} + a_{30}, a_{40} = a_{21}(a_{26}a_{33} - a_{27}) + a_{30}(a_{23} - a_{19}) + a_4 a_{25} a_{26} - a_9 a_{21} a_{28} - a_4 a_9 a_{30} - a_{25} a_{28}, a_{41} = -a_{21}(a_{22}a_{28} + a_{23}a_{27}) - a_{25}(a_{24}a_{27} - a_{19}a_{28}) - a_{30}(a_{19}a_{23} - a_4 a_{22}), a_{42} = a_{29} - a_{23} + a_{19} + a_4 a_9 + a_{20} a_{26}, a_{43} = a_{23}a_{29} - a_{24}a_{28} - a_{19}(a_{29} - a_{23}) - a_4(a_{29}a_9 - a_{22} - a_{24}a_{26}) + a_{20}(a_{23}a_{26} + a_{27} - a_9 a_{28}), a_{44} = a_{19}(a_{24}a_{28} - a_{23}a_{29}) - a_4(a_{22}a_{29} + a_{24}a_{27}) - a_{20}(a_{27}a_{23} + a_{28}a_{22}),$$

Appendix B

$$\Delta_{1i} = \begin{vmatrix} m_i^2 + a_{23} & a_{24} & a_{25} \\ a_{28} & a_{29} - m_i^2 & a_{30} \\ 0 & a_{31}m_i^2 + a_{32} & a_{33}m_i^2 + a_{34} \end{vmatrix}, \Delta_{2i} = \begin{vmatrix} a_9 m_i^2 + a_{22} & a_{24} & a_{25} \\ a_{26}m_i^2 - a_{27} & a_{29} - m_i^2 & a_{30} \\ (m_i^2 - k^2)^2 & a_{31}m_i^2 + a_{32} & a_{33}m_i^2 + a_{34} \end{vmatrix},$$

$$\Delta_{3i} = \begin{vmatrix} a_9 m_i^2 + a_{22} & m_i^2 + a_{23} & a_{25} \\ a_{26}m_i^2 - a_{27} & a_{28} & a_{30} \\ (m_i^2 - k^2)^2 & 0 & a_{33}m_i^2 + a_{34} \end{vmatrix}, \Delta_{4i} = \begin{vmatrix} a_9 m_i^2 + a_{22} & m_i^2 + a_{23} & a_{24} \\ a_{26}m_i^2 - a_{27} & a_{28} & a_{29} - m_i^2 \\ (m_i^2 - k^2)^2 & 0 & a_{31}m_i^2 + a_{32} \end{vmatrix},$$

Appendix C

$$b_1 = \frac{\lambda_0}{\rho c_1^2}, b_2 = \frac{\lambda}{\rho c_1^2}, b_3 = \frac{2\mu + K}{\rho c_1^2}, b_5 = \frac{\mu + K}{\rho c_1^2}, b_6 = \frac{\mu}{\rho c_1^2}, b_7 = \frac{K}{\rho c_1^2}, b_8 = \frac{\omega^{*2}\gamma}{\rho c_1^4}, b_9 = \frac{\omega^{*2} b_0}{\rho c_1^4}, b_{10} = \frac{\omega^{*2}}{\rho c_1^4}$$

$$g_{1i} = \alpha_{1i} + (m_i^2 - b_2 k^2) - \alpha_{2i} + b_{11}\alpha_{3i}, g_{2i} = -\iota b_3 k m_i, g_{3i} = \iota b_9 k \alpha_{1i},$$

$$g_{4i} = -\alpha_0 b_{10} m_i \alpha_{1i}, g_{5i} = \alpha_{2i}, g_{6i} = \alpha_{3i}, \qquad i = 1,2,3,4$$

$$g_{1i} = \iota b_3 k m_i, g_{2i} = (b_6 m_i^2 + b_5 k^2) - b_7 \alpha_{4i}, g_{3i} = -b_8 \alpha_{4i} m_i,$$

$$g_{4i} = -ikb_0 b_{10}\alpha_{4i}, g_{5i} = 0, g_{6i} = 0, \qquad i = 5,6$$

$$M_1 = \left(\frac{b_1 f_2 + (\gamma^{*2} - b_2 k^2) f_1 - f_3 + b_{11} f_4}{f_5}\right), M_2 = \frac{-\iota b_3 k \gamma^* f_1}{f_5}, M_3 = \frac{\iota b_9 k f_2}{f_5}, M_4 = \frac{-\alpha_0 b_{10} \gamma^* f_2}{f_5}, M_5 = \frac{f_3}{f_5}, M_6 = \frac{f_4}{f_5}$$