Thermodynamical Behavior of Laser Irradiated Mass Diffusive Micro Stretch Thermoelastic Medium

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Abstract

This paper is concerned with the elastodynamic interactions of the ultra-laser heat source with homogeneous micro stretch-thermoelastic mass diffusion medium. The medium is under application of various forces. Normal mode analysis technique has been applied to the basic equations to solve the problem. Expressions are derived for normal stress, tangential stress, micro stress and temperature distribution. The numerically computed results have graphs also. The analysis of various stress quantities is there in this model. This research has some special cases from the present investigation.

Index terms— mass diffusion, micro stretch-thermoelastic, laser heat source, normal mode analysis, normal force.

1 Introduction

ringen **??**1] developed the theory of thermo-micro stretch elastic solids. Micro stretch continuum is a model for Bravais lattice with basis on the atomic level and two-phase dipolar solids with a core on the macroscopic level. Composite materials reinforced with chopped elastic fibers, porous media with pores containing gas or in viscid liquid, asphalt or other elastic inclusions and solid-liquid crystals etc. are examples of micro stretch solids. **??**zzat et al. [2,3] discussed the concept of thermal relaxation. Marin [4,5] investigated various problems in micropolar thermoelasticity and micro stretch thermoelasticity.

Diffusion is the spontaneous movement of the particles from a high concentration region to the lowconcentration region, and it occurs in response to a concentration gradient expressed as the change in the concentration due to change in position. Thermal diffusion utilizes the transfer of heat across a thin liquid or gas to accomplish isotope separation. Simply concentration is calculated using Fick's law. This law does not consider the mutual interaction between the inclusion substance and the medium. The thermo diffusion in elasticity is result of the coupling of temperature, mass diffusion and that of strain in addition to heat and mass exchange with the environment. Nowacki [6][7][8][9] developed the theory of thermoelastic diffusion by using coupled thermoelastic model. Dudziak and Kowalski [10] and Olesiak and Pyryev [11], respectively, discussed the theory of thermo diffusion and coupled quasi-stationary problems of thermal diffusion for an elastic layer.

Thermal shock due to exposure to an ultra-short laser pulse is interesting from the point of thermo elasticity since they require a coupled analysis of the temperature and deformation fields. A thermal shock induces very rapid movement in the structural elements, giving rise to very significant inertial forces, and thereby, an increase in vibration. In irradiation of ultra-short pulsed laser, the high-intensity energy flux and ultrashort duration lead to a very high thermal gradient. So, in these cases, Fourier law of heating is no longer valid. Scruby et al. [12] and Rose [13] considered the point source model of lasers. Later McDonald [14] and Spicer [15] proposed a new model known as laser-generated ultrasound model by introducing the thermal diffusion effect. Dubois [16] experimentally demonstrated that penetration depth plays an important role in the laserultrasound generation process. The thermoelastic response of laser in context of four theories was discussed by Youssef and Al-Bary [17]. A problem for a thick plate under the effect of laser pulse thermal heating was studied by Elhagary [18]. Kumar et al. [19] studied the thermo-mechanical interactions of a laser pulse with the micro stretch thermoelastic medium.

This present research deals with the disturbance in a homogeneous micro stretch thermoelastic medium with mass diffusion due to the effect of an ultra-laser heat source. The normal mode analysis technique is used to obtain the expressions for the displacement components, couple stress, temperature, mass concentration and micro stress distribution due to various sources.

2 II. Basic Equations

Following Eringen [20] and Al-Qahtani and Datta [21], the basic equations for homogeneous micro stretch thermoelastic mass diffusion medium in the absence of body force, body couple with laser heat source are given by: a) Stress equation of motion(?? + ??)?(?. ??) + (?? + ??)? 2 ?? + ??? × ?? + ?? 0 ??? * ? ?? 1 ?1 + ?? 1 ?? ???? ? ??? ? ?? 2 ?1 + ?? 1 ?? ???? ? ??? = ????? (1) b) Couple stress equation of motion (??? 2 ? 2??)?? + (?? + ??)?(?. ??) + ??? × ?? = ?????? ? (2)

c) The equation of balance of stress moments(?? 0 ? 2 ? ? 1)?? * ? ?? 0 ?. ?? + ?? 1 ?1 + ?? 1 ?? ???? ? ?? + ?? 2 ?1 + ?? 1 ?? ???? ? ?? = ???? 0 2 ?? * ?**(3)**

d) The equation of heat conduction?? * ? 2 ?? = ???? * ? ?? ???? + ?? 0 ?? 2 ???? 2 ? ?? + ?? 1 ?? 0 ? ?? ???? + ???? 0 ?? 2 ???? 2 ? (?. ?? ? ??) + ?? 1 ?? 0 ? ?? ???? + ???? 0 ?? 2 ???? 2 ? ?? * + ???? 0 ? ?? ???? + ?? 1 ?? 2 ???? 2 ? ?? (4)

e) The equation of mass diffusion is???? 2 ? 2 (?. ??) + ???? ?1 + ?? 1 ?? ???? ? ? 2 ?? + ? ?? ???? + ???? 0 ?? 2 ???? 2 ? ?? ? ???? ?1 + ?? 1 ?? ???? ? ? 2 ?? = 0**(5)**

f) The constitutive relations are?? ???? = ??? 0 ?? * + ???? ??,?? ??? ???? + ????? ??,?? + ?? ?? ,?? ? + ????? ?? ,?? ? ?? ?????? ?? ?? ? ? ?? 1 ?1 + ?? 1 ?? ???? ? ?? ???? ?? ? ?? 2 ?1 + ?? 1 ?? ???? ? ?? ???? ??**(6)**

?? ???? = ???? ??,?? ?? ???? + ???? ??,?? + ???? ?? ,?? + ?? 0 ?? ?????? ?? ,?? *?? ?? * = ?? 0 ?? ,?? * + ?? 0 ?? ?????? ?? ?? ,??**(7)**

The plate surface is illuminated by laser pulse given by the heat input?? = ?? 0 ð ??"ð ??"(??)ð ??"ð ??"(?? 1)?(?? 3)**(9)**

Where ?? 0 is the energy absorbed. The temporal profile ð ??"ð ??"(??) is represented as,ð ??"ð ??"(??) = ?? ?? 0 2 ?? ?? ?? ?? 0 ? (10)

Here ?? 0 is the pulse rise time. The pulse is also assumed to have a Gaussian spatial profile in ?? 1ð ??"ð ??"(?? 1) = 1 2???? 2 ?? ?? ?? 1 2 ?? 2 ? (**11**)

Where ?? is the beam radius, and as a function of the depth ?? 3 the heat deposition due to the laser pulse is assumed to decay exponentially within the solid,?(?? 3) = ?? * ?? ??? * ?? 3**(12)**

Equation (9) with the aid of (10-11) and (12) takes the form:?? = ?? 0 ?? * 2???? 2 ?? 0 2 ???? ?? ?? ?? 0 ? ?? ?? ?? 1 2 ?? 2 ? ?? ??? * ?? 3 , (**13**) t f(t) x 1 g(x 1) x 3 h(x 3)

Fig. **??**: Temporal profile of ð ??"ð ??"(??). Fig. **??**: Profile of ð ??"ð ??"(?? 1). Fig. **??**: Profile of ?(?? 3).

Here ??, µ, ??, ??, ??, ??, ?? 0 , ?? 1 , ?? 0 , ?? 0 , are material constants, ?? is mass density, ?? = (?? 1 , ?? 2 , ?? 3)is the displacement vector and ?? = (?? 1 , ?? 2 , ?? 3)is the microrotation vector, ?? * is the scalar micro stretch function, ?? is temperature and ?? 0 is the reference temperature of the body chosen,?? is the concentration of the diffusion material in the elastic body,?? * is the coefficient of the thermal conductivity,?? * is the specific heat at constant strain, ?? is the thermoelastic diffusion constant, ?? is the coefficient describing the measure of thermo diffusion and ?? is the coefficient describing the measure of mass diffusion effects, ?? is the microinertia, ?? 1 = (3?? + 2?? + ??)?? ??1 ,?? 2 = (3?? + 2?? + ??)?? ??1 , ?? 1 = (3?? + 2?? + ??)?? ??2 , ?? 2 = (3?? + 2?? + ??)?? ??2 ,

?? ??1 , ?? ??2 are coefficients of linear thermal expansion and ?? ??1 , ?? ??2 are coefficients of linear diffusion expansion, ?? 0 is the microinertia for the microelements,?? ???? are components of stress, ?? ???? are components of couple stress, ?? ?? * is the micro stress tensor, ?? ???? are components of strain, ?? ???? is the dilatation,?? ???? is Kroneker delta function, ?? 0 , ?? 1 are the diffusion relaxation times and ?? 0 , ?? 1 are thermal relaxation times with ?? 0 ? ?? 1 ? 0.

In the above equations symbol (",") followed by a suffix denotes differentiation with respect to spatial coordinates and a superposed dot (" ? ") denotes the derivative with respect to time respectively.

3 III.

4 Formulation of the Problem

We consider a micro stretch thermoelastic mass diffusion medium with rectangular Cartesian coordinate system ???? 1 ?? 2 ?? 3 with ?? 3 -axis pointing vertically downward the medium.

5 Laser pulse

?? 2 ?? 3 = 0 ?? 1 Micro stretch Thermoelastic Mass Diffusion Medium (0 ? ?? 3 < ?) ?? 3

6 Fig. 4: Geometry of the Problem

For two dimensional problems, we take the displacement vector and micro rotation vector as:?? = (?? 1 , 0, ?? 3),?? = (0, ?? 2 , 0),**(14)** For further consideration it is convenient to introduce in equations (**??**)-(5) the dimensionless quantities defined by:?? ?? ? = ?? ð ??"ð ??" * ?? 1 ?? 1 ?? 0 ?? ?? , ?? ?? ? = ð ??"ð ??" * ?? 1 ?? ?? , ?? ? = ð ??"ð ??" * ?? , ?? ? = ?? ?? 0 , ?? 1 ? = ð ??"ð ??" * ?? 1 , ?? 0 ? = ð ??"ð ??" * ?? 0 ,?? 1 ? = ð ??"ð ??" * ?? 1 , ?? ???? ? = 1 ?? 1 ?? 0 ?? ???? , ð ??"ð ??" * = ???? * ?? 1 2 ?? * , ?? ?? ? = ???? 1 2 ?? 1 ?? 0 ?? ?? , ?? 1 ? = ð ??"ð ??" * ?? 1 , ?? 1 2 = ??+2?? +?? ?? , ?? 2 2 = ?? +?? ?? , ?? 3 2 = ?? ???? , ?? 4 2 = 2?? 0 ?? ?? 0 , ?? = ?? 2 ?? 0 ?? 2 ?? * ?? 1 ,?? ???? * = ð ??"ð ??" * ???? 1 ?? 0 ?? ???? , ?? ? = ?? 2 ???? 1 2 ?? , ?? = ?? * ð ??"ð ??" * 2 ?? * ?? ? , ?? * ? = ?? ?? 1 2 ?? 1 ?? 0 ?? *(**15**)

By Helmholtz representation of a vector into scaler and vector potentials the displacement components ?? 1 and ?? 3 are related to non-dimensional potential functions ?? and ð ??"ð ??" as:?? 1 = ???? ???? 1 ? ??ð ??"ð ??" ???? 3 , ?? 3 = ???? ???? 3 + ??ð ??"ð ??" ???? 1 (16)

Substituting the values of ?? 1 &?? 3 from (**??**6) in (1)-(5) and with the aid of (14) & (15), after suppressing the primes, we obtain:? 2 ?? ? ?? ?+ ?? 4 ?? * ? ?1 + ?? 1 ?? ???? ? ?? ? ?? 5 ?1 + ?? 1 ?? ???? ? ?? = 0,(**17**) ?? 2 ? ?? 8 ? ?? 12 ?? 2 ???? 2 ? ?? * ? ?? 9 ? 2 ?? + ?? 10 ?1 + ?? 1 ?? ???? ? ?? + ?? 11 ?1 + ?? 1 ?? ???? ? ?? = 0,(**18**)? ?? ???? + ?? 0 ?? 2 ???? 2 ? ? 2 ? ?? + ?1 + ???? 0 ?? ???? ? ??? 13 ? 2 ?? + ?? 14 ?? * ?? + ?? 15 ?1 + ?? 1 ?? ???? ? ?? ?= ?? 0 ð ??"ð ??" * (?? 1 , ??)?? ??? * ?? 3 ,(**19**)? 4 ?? + ?? 16 ?1 + ?? 1 ?? ???? ? ? 2 ?? + ?? 17 ? ?? ???? + ???? 0 ?? 2 ???? 2 ? ?? ? ?? 18 ?1 + ?? 1 ?? ???? ? ? 2 ?? = 0 (20) ?? 2 ? 2 ð ??"ð ??" ? ð ??"ð ??" ?+ ?? 3 ?? 2 = 0,(**21**)? 2 ?? 2 ? 2?? 6 ?? 2 ? ?? 6 ? 2 ? = ?? 7 ?? ?2,(**22**)Here? 2 = ?? 2 ???? 1 2 + ?? 2 ???? 3 2 is Laplacian operator, ð ??"ð ??"(?? 1 , ??) = ??? + ???? 0 ?1 ? ?? ?? 0 ?? ?? ?? ?? 1 2 ?? 2 + ?? ?? 0 ? and ?? 0 = ?? 20 ?? 0 ?? * 2???? 2 ?? 0 2 IV.

7 Solution of the Problem

The solution of the considered physical variables can be decomposed in terms of the normal modes as in the following form:{??, ð ??"ð ??", ??, ?? 2 , ?? * , ??}(?? 1 , ?? 3 , ??) = {?? ? , ð ??"ð ??" ? , ?? ? , ?? 2 ???? , ?? * ???? , ?? ? }(?? 3)?? ??(???? 1 ?ð ??"ð ??"??) (23)

Here ð ??"ð ??" is the angular frequency and ?? is wave number.

Making use of (23), equations (17)-(22) after some simplifications yield:[???? 8 + ???? 6 + ???? 4 + ???? 2 + ??]?? ? = ð ??"ð ??" 1 (?? * , ?? 1 , ??)?? ??? * ?? 3(**24**)

[???? 8 The solution of the above system of equations (24)-(28) satisfying the radiation conditions that (?? ? , ð ??"ð ??" ? , ?? ? , ?? 2 ???? , ?? ?) ? 0 as ?? 3 ? ? are given as following:?? ? = ? ?? ?? ?? ??? ?? ?? 3 4 ??=1 + ð ??"ð ??" 1 ð ??"ð ??" 5 ?? ??? * ?? 3 (29) ?? * ???? = ? ?? 1?? ?? ?? ?? ??? ?? ?? 3 4 ??=1 + ð ??"ð ??" 2 ð ??"ð ??" 5 ?? ??? * ?? 3 (30) ?? ? = ? ?? 2?? ?? ?? ?? ??? ?? ?? 3 4 ??=1 + ð ??"ð ??" 3 ð ??"ð ??" 5 ?? ??? * ?? 3 (31) ?? ? = ? ?? 3?? ?? ?? ?? ??? ?? ?? 3 + ð ??"ð ??" 4 ð ??"ð ??" 5 ?? ??? * ?? 3 4 ??=1 , (**32**) (ð ??"ð ??" ? , ?? 2 ????) = ? (1, ?? ??)?? ?? ?? ??? ?? ?? 3 6 ??=5 ,(**33**)

Where?? ?? 2 (?? = 1,2,3,4) are the roots of the equation (24) and ?? ?? 2 (?? = 5,6) are the roots of characteristic equation of equation (28) and?? 1?? = ? Î?" 2?? Î?" 1?? , ?? 2?? = Î?" 3?? Î?" 1?? , ?? 3?? = ? Î?" 4?? Î?" 1?? , ?? = 1,2,3,4&?? ?? = ?? 3 ??? 2 ?? ?? 2 +?? 35 ? , ?? = 5,6

Here,Î?" 1?? , Î?" 2?? , Î?" 3?? , Î?" 4?? are defined in Appendix B.

Substituting the values of ?? ? , ?? * ???? , ?? ? , ð ??"ð ??" ? , ?? ? 2 , ?? ? from the equations (**??**9)-(33) in the (6)-(8), and using (14)-(**??**6) & (23) and then solving the resulting equations, we obtain: ?? ? 33 = ? ?? 1?? ?? ??? ?? ?? 3 6 ??=1 ? ?? 1 ?? ??? * ?? 3 (34) ?? ? 31 = ? ?? 2?? ?? ??? ?? ?? 3 6 ??=1 ? ?? 2 ?? ??? * ?? 3 , (**35**) ?? ? 32 = ? ?? 3?? ?? ??? ?? ?? 3 6 ??=1 ? ?? 3 ?? ??? * ?? 3 (36) ?? 3 * = ? ?? 4?? ?? ??? ?? ?? 3 ? ?? 4 ?? ??? * ?? 3 , 6 ??=1(**37**)?? ? = ? ?? 5?? ?? ??? ?? ?? 3 ? ?? 5 ?? ??? * ?? 3 6 ??=1 ,(**38**)?? ? = ? ?? 6?? ?? ??? ?? ?? 3 5 ??=1 ? ?? 6 ?? ??? * ?? 3 ,(**39**)

8 Boundary Conditions

We consider normal force and thermal and mass concentration sources are acting at the surface ?? 3 = 0 along with vanishing of couple stress in addition to thermal and mass concentration boundaries considered at?? 3 = 0 and?? 0 = 0 . Mathematically this can be written as:?? 33 = ??? 1 ?? ??(???? 1 ?ð ??"ð ??"??) , ?? 31 = 0, ?? 32 = 0, ?? 3 * = 0, ???? ???? 3 = ?? 2 ?? ??(???? 1 ?ð ??"ð ??"??) , ???? ???? 3 = ?? 3 ?? ??(???? 1 ?ð ??"ð ??"??) (40)

Where ?? 1 and ?? 2 are the magnitude of the applied force. Substituting the expression of the variables considered into these boundary conditions, we can obtain the following system of equations:? (?? 1?? , ?? 2?? , ?? 3?? , ?? 4?? , ?? ?? ?? 5?? , ?? ?? ?? 6??)?? ?? = (??? 1 , 0,0,0, ??? 2 , ??? 3) 6 ??=1

(41)

The system of equations (**??**1) is solved by using the matrix method as follows: ? ? ? ? ? ? ?? 1 ?? 2 ?? 3 ?? 4 ?? 5 ?? 6 ? ? ? ? ? ? = ? ? ? ? ? ? ð ??"ð ??"? ? ? ? ? ? ?1 ? ? ? ? ? ? ??? 1 0 0 0 ??? 2 ??? 3 ? ? ? ? ? ? (42) VI.

9 SPECIAL CASES VII. Numerical Results and Discussions

The analysis is conducted for a magnesium crystal-like material. The values of constants are as: ?? = 9.4 × 10 10 ???? ?2 ,?? = 4.0 × 10 10 ???? ?2 , ?? = 1.0 × 10 10 ???? ?2 , ?? = 1.74 × 10 3 ??ð ??"ð ??"?? ?3 , ?? = 0.2 × 10 ?19 ?? 2 , ?? = 0.779 × 10 ?9 ?? Thermal, diffusion and micro stretch parameters are given by:

A comparison of the dimensionless form of the field variables for the cases of micro stretch thermoelastic mass diffusion medium with a laser pulse (MTMDL), micro stretch thermoelastic mass diffusion medium without a laser pulse (MTMD) subjected to normal force is presented in Figures **??**-13. The values of all physical quantities for both cases are shown in the range 0 ? ?? 3 ? 5.?? * = 1.04 × 10 3 ????ð ??"ð ??" ?1 ?? ?1 , ?? * = 1.7 × 10 6 ???? ?1 ?? ?1 ?? ?1 , ?? ??1 = 2.33 × 10 ?5 ?? ?1 , ?? ??2 = 2.48 × 10 ?5 ?? ?1 , ?? 0 = 0.298 × 10 3 ??, ?? 0 = 0.02, ?? 1 = 0.01, ?? ??1 = 2.65 × 10 ?4 ?? 3 ??ð ??"ð ??" ?1 , ?? ??2 = 2.83 × 10 ?4 ?? 3 ??ð ??"ð ??" ?1 , ?? = 2.9 × 10 4 ?? 2 ?? ?2 ?? ?1 , ?? = 32 × 10 5 ??ð ??"ð ??" ?1 ?? 5 ?? ?2 , ?? 1 = 0.

Solid lines, dash lines corresponds to micro stretch thermoelastic mass diffusion medium with laser pulse (MTMDL) and micro stretch thermoelastic mass diffusion medium without laser pulse (MTMD) respectively. The computations were carried out in the absence and presence of laser pulse (?? 0 = 10 5 , 0) and on the surface of plane ?? 1 = 1, ?? = 0.1 Fig. **??** shows the variation of normal stress ?? 33 with the distance ?? 3 . It is noticed that for MTMDL and MTMD, the normal stress ?? 33 show similar behavior. The normal stress in both the cases initially increases and then monotonically decreases. The value of ?? 33 increases near the application of the normal force due to the stretch effect and then decreases. Fig. **??** shows the variation of couple stress ?? 32 with distance ?? 3 for MTMDL and MTMD. The variation of ?? 32 for (MTMDL, MTMD) is monotonically decreasing in region 0 ? ?? 3 ? 1 and monotonically increasing after that. The ?? 32 approaches to zero away from the point of application of source. It is clear from figure **??** that laser source has a significant effect on the value of?? 32 .

Fig. **??** depicts the variation of micro stress ?? 3 * with distance. The variation of ?? 3 * is similar for both the cases in the beginning and in the last, however ?? 3 * for MTMD show oscillatory behavior in range1 ? ?? 3 ? 4 Fig. **??**0 show variation of mass concentration w.r.t. distance?? 3 . Mass concentration monotonically decreases with increasing distance from application of source. The laser source seems to have no significant effect on variation of mass concentration.

10 Conclusions

The problem consists of investigating displacement components, scalar micro stretch, temperature distribution and stress components in a micro stretch thermoelastic mass diffusion medium subjected to input laser heat source. Normal mode analysis is employed to express the results. Theoretically obtained field variables are also depicted graphically.

The analysis of results permits some concluding remarks: 1) It is clear from the figures that all the field variables have nonzero values only in the bounded region of space indicating that all the results are in agreement with the various theories of thermoelasticity.

2) The effect of the input laser heat source is much pronounced in normal stress, tangential stress, micro stress, temperature distribution and displacement components. Change in the value of ?? ?? cause significant changes in all these simulated resulting quantities. 3) It is noticed from the figures that the laser heat source has no significant role on mass concentration. 4) The trend of variation of physical quantities show similarity with Elhagary [18] although effect is included.

11 References Références Referencias

Appendix A ??

1 = ?? + ?? ?? 1 ?? 0 , ?? 2 = ?? + ?? ?? 1 ?? 0 , ?? 3 = ?? ???? 1 2 , ?? 4 = ?? 0 ???? 1 2 , ?? 5 = ???? 1 2 ?? 1 ?? 0 , ?? 6 = ???? 1 2 ??ð ??"ð ??" * 2 , ?? 7 = ?????? 1 2 ?? , ?? 8 = ?? 1 ?? 1 2 ?? 0 ð ??"ð ??" * 2 , ?? 9 = ?? 0 ???? 1 4 ?? 1 ?? 0 ?? 0 ð ??"ð ??" * 2 , ?? 10 = ?? 1 ???? 1 4 ?? 1 ?? 0 ð ??"ð ??" * 2 , ?? 11 = ?? 2 ?? 2 ?? 1 6 ?? 1 ?? 2 ?? 0 ?? 0 ð ??"ð ??" * 2 , ?? 12 = ???? 1 2 ?? 0 2?? 0 , ?? 13 = ?? 0 ?? 1 2 ??ð ??"ð ??" * ?? * , ?? 14 = ?? 1 ?? 1 ?? 0 ??ð ??"ð ??" * ?? * , ?? 15 = ?????? 1 4 ð ??"ð ??" * ?? 2 ?? * , ?? 16 = ?????? 1 2 ?? 1 ?? 2 , ?? 17 = ???? 1 4 ð ??"ð ??" * ???? 2 2 , ?? 18 = ?????? 1 2 ?? 2 2 , ?? 20 = ???? 1 4 ð ??"ð ??" * 2 ?? 1 ?? * , ?? 19 = ð ??"ð ??" 2 ? ?? 2 , ?? 20 = 1 ? ??ð ??"ð ??"?? 1 , ?? 21 = ?? 5 (1 ? ??ð ??"ð ??"?? ?), ?? 22 = ?? 2 ?? 9 , ?? 23 = ð ??"ð ??" 2 ?? 12 ? ?? 8 ? ?? 2 , ?? 24 = ?? 10 (1 ? ??ð ??"ð ??"?? 1), ?? 25 = ?? 11 (1 ? ??ð ??"ð ??"?? ?), ?? 26 = ??? 13 (??ð ??"ð ??" + ð ??"ð ??" 2 ???? 0), ?? 27 = ?? 2 ?? 26 , ?? 32 = ??? 2 ?? 31 , ?? 28 = ??? 14 (??ð ??"ð ??" + ð ??"ð ??" 2 ???? 0), ?? 29 = ?? 2 ? ??ð ??"ð ??" ? ð ??"ð ??" 2 ?? 0 , ?? 30 = ??? 15 (??ð ??"ð ??" + ð ??"ð ??" 2 ?? 1), ?? 31 = ?? 16 (1 ? ??ð ??"ð ??"?? 1), ?? 33 = ??? 18 (1 ? ??ð ??"ð ??"?? ?), ?? 34 = ?? 17 (??ð ??"ð ??" ? ð ??"ð ??" 2 ???? 0), ?? 35 = ð ??"ð ??" 2 ? ?? 2 ?? 2 , ?? 36 = ð ??"ð ??" 2 ?? 7 ? ?? 2 ? 2?? [1] [2]

(27)

[?? 4 + ???? 2 + ??]ð ??”ð ??” ? = 0 (28)

Where ?? = ?? ???? 3

Figure 1: ,

11	ð ??”ð ??” 12	ð ??”ð ??” 13	ð ??”ð ??” 14	ð ??”ð ??” 15	ð ??”ð ??” 16
ð ??”ð ??” 21	ð ??”ð ??” 22	ð ??”ð ??” 23	ð ??”ð ??” 24	ð ??”ð ??” 25	ð ??”ð ??” 26
ð ??”ð ??” 31	ð ??”ð ??” 32	ð ??”ð ??” 33	ð ??”ð ??” 34	ð ??”ð ??” 35	ð ??”ð ??” 36
ð ??”ð ??” 41	ð ??”ð ??” 42	ð ??”ð ??” 43	ð ??”ð ??” 44	ð ??”ð ??” 45	ð ??”ð ??” 46
?? 1					

Figure 2:

Appendix B

?? ?? ?? 24 ?? 25
2 + ??
23
? 1?? = ? ?? 28 0 ?? 29 ? ?? ?? 2 ?? 31 ?? ?? 2 + ?, ?? 30
?? 9 ?? ?? 2 + ?? 22 ?? ?? 2 + ?? 23 ?? 25

? 3?? = ? ?? 26 ?? ?? 2 ? ?? 27 ?? ?? 30 ? , ? 4?? = ?
28

(?? ?? 2 ? ?? 2) 2 0 ?? 33 ?? ?? 2 + ?? 34

Appendix C

?? ?? 0 ?? ???? 1 2 , ?? 3 = 2?? , ?? + ?? ???? 1 2 , ?? 6 = ?? ???? 1 2 , ?? 7 =
1 ???? + ?? ??
= 1 2 , ???? 1 5
?? 2 2 =
=

ð ??”ð ??” ? , ?? 2 = ????? 3 ???? * ð ??”ð ??” 1 ð ??”ð ??” 5 , ?
5

Figure 3:

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